

The $\Lambda_c(2595)$ resonance as a dynamically generated state: the compositeness condition and the large N_c evolution

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Abstract

Recent studies have shown that the well established $\Lambda_c(2595)$ resonance contains a large meson-baryon component, which can vary depending on the specific formalism. In this work, we examine such a picture by utilizing the compositeness condition and the large number of colors (N_c) expansion. We examine three different models fulfilling two body unitarity in coupled-channels, and adopting renormalization schemes where the mass of the $\Lambda_c(2595)$ resonance is well described, but not necessarily its width, since we do not consider three body channels and work at the isospin symmetric limit. Both approximations might have an effect larger on the width than on the mass. In this context, our studies show that the compositeness of the $\Lambda_c(2595)$ depends on the number of considered coupled channels, and on the particular regularization scheme adopted in the unitary approaches and, therefore, is model dependent. In addition, we perform an exploratory study of the $\Lambda_c(2595)$ in the large N_c expansion, within a scheme involving only the $\pi\Sigma_c$ and $K\Xi'_c$ channels, whose dynamics is mostly fixed by chiral symmetry. In this context and formulating the leading-order interaction as a function of N_c , we show that for moderate $N_c > 3$ values, the mass and width of the $\Lambda_c(2595)$ deviate from those of a genuine qqq baryon, implying the relevance of meson-baryon components in its wave function. Furthermore, we study the properties of the $\Lambda_c(2595)$, in the strict $N_c \rightarrow \infty$ limit, using an extension of the chiral Weinberg-Tomozawa interaction to an arbitrary number of flavors and colors. This latter study hints at the possible existence of a (perhaps) sub-dominant qqq component in the $\Lambda_c(2595)$ resonance wave function, which would become dominant when the number of colors gets sufficiently large.

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I. INTRODUCTION

In the naive quark model, mesons are made up of a quark-antiquark pair while baryons consist of three quarks. Before 2000, most hadrons could be easily understood within such a picture, with the exception of only a few cases, e.g., the lowest lying scalar nonet, the $\Lambda(1405)$, and the Roper resonances [1]. The situation changed dramatically in 2003 with the discovery of the $X(3872)$ by the BELLE collaboration [2], that was the first of many others, so-called XYZ states, which could not be easily accommodated into standard models of constituent quarks. Indeed, some of them apparently contain more than the minimum quark content dictated by the naive quark model, such as the $Z_c(4430)$ [3] and $Z_c(3900)$ [4]. The latest P_c states discovered by the LHCb collaboration [5] are the first exotic states of such type in the baryonic sector. Various theoretical interpretations of these resonances have been proposed, ranging from weakly bound molecular or compact multi-quark states to quark-gluon hybrids. As many of these exotic states are located close to the two- or even three-body strong decay thresholds, coupled-channel effects are widely believed to play an important role.

Unitarized approaches and their extensions, which take into account various important constraints, such as chiral and heavy quark symmetries, or unitarity, provide a useful framework to study coupled-channel effects. In certain cases, the interactions among the coupled channels can be strong enough to generate the so-called dynamically generated states, which are customarily referred to as molecular states as well. It is found that, somehow unexpectedly, not only the exotic states, but also some states long believed to be conventional hadrons, which can be explained by the constituent quark models, turn out to contain large hadron-hadron components. Some of the prominent examples are the axial vector mesons [6–9] and the low-lying tensor states [8–11]. Many studies of these states in various decays and reactions have been performed and all the results seem to be consistent with such a molecular picture.

In the heavy-flavor baryon sector, the $1/2^- \Lambda_c(2595)$ and its heavy quark symmetry (HQS) partners have been proposed to be of molecular nature as well, although there is debate about its most important components [12–18]. More specifically in Refs. [12, 13], it is claimed that $\pi\Sigma_c$ channel plays the dominant role, while DN is found to be the most important ingredient in Ref. [14].¹ After including the DN and D^*N channels, as required by heavy quark spin symmetry (HQSS) arguments, the authors of Refs. [15, 16, 18] conclude that both of them may be needed.

¹ A similar conclusion was reached in the Jülich meson-exchange model [19].

In principle, wave functions are not observables themselves. As a result, it is difficult to pin down the exact nature of a hadronic state. The claims regarding the largest Fock components in hadron wave functions are often model dependent. In recent years, the compositeness condition, first proposed by Weinberg to explain the deuteron as a neutron-proton bound state [20, 21], has been advocated as a model independent way to determine the relevance of hadron-hadron components in a molecular state. With renewed interests in hadron spectroscopy, this method has been extended to more deeply bound states, resonances, and higher partial waves [22–35]. For the particular case of the $\Lambda_c(2595)$, the situation is a bit unclear. For instance, it was shown in Ref. [36] that the $\Lambda_c(2595)$ is not predominantly a $\pi\Sigma_c$ molecular state using the effective range expansion. A similar conclusion was reached in Ref. [37], using a generalized effective range expansion including Castillejo-Dalitz-Dyson pole contributions. In this latter work, the effects of isospin breaking corrections are also taken into account and the extended compositeness condition for resonances developed in Ref. [38] has been applied to calculate the compositeness coefficients. Furthermore, although in the unitary approaches [12–16, 18] the $\Lambda_c(2595)$ is found to be of molecular nature, there is no general agreement on its dominant meson-baryon components yet.

Another approach ² to probe the dominant component of a hadronic state is to study the N_c dependence of the poles associated to resonances that appear in the unitarized meson-meson [45–54] or meson-baryon [55–58] scattering amplitudes, being N_c the number of colors of quarks. The $1/N_c$ expansion [59–64] is valid for the whole energy region and makes specific predictions for $q\bar{q}$ and qqq states. A genuine $q\bar{q}$ state becomes bound as $N_c \rightarrow \infty$ with its mass scaling as $\mathcal{O}(1)$ and its width as $\mathcal{O}(1/N_c)$. Mesonic states of other nature may show different behavior [65]. The mass of a generic qqq state with two or three flavors evolves as $\mathcal{O}(N_c)$ while its width scales as $\mathcal{O}(1)$ at leading order [60, 66, 67].

In the present work, we utilize both the compositeness condition and the large N_c behaviour to examine the nature of the $\Lambda_c(2595)$ aiming to test the molecular scenario. This paper is organized as follows. In Sect. II, we briefly introduce the unitarized models used in Refs. [12, 15, 18]. In Sect. III, we discuss the compositeness condition and, in particular, the effects due to the number of coupled channels considered and to the specific regularization scheme adopted. In Sect. IV, we formulate the large N_c expansion within the model of Ref. [12], and show that in this scheme, and for a moderately large number of colors, the N_c dependence of the $\Lambda_c(2595)$

² In recent years, it has been stressed that the quark mass dependence of a hadronic state, which can be accessed by present lattice QCD simulations, can also be used to distinguish its nature. In the present work, we are not going to approach the problem from this perspective. Interested readers can see, e.g., Refs. [39–44] and references therein.

mass and width deviates from that of a genuine qqq state. We will also discuss the $N_c \gg 3$ behavior of the $\Lambda_c(2595)$ pole position within the dynamical model established in [15], using the findings of Refs. [55, 56], where the chiral Weinberg-Tomozawa (WT) interaction is extended to an arbitrary number of flavors and colors. This latter study hints at the possible existence of a (perhaps) sub-dominant qqq component in the $\Lambda_c(2595)$ resonance wave function, which would become dominant when the number of colors gets sufficiently large. Finally, the most relevant conclusions of this work are collected in Sect. V.

II. UNITARIZED APPROACHES

The key ingredients of unitary approaches are kernel potentials and the procedures adopted to restore exact two-body unitarity in coupled channels. In practice, the kernel potentials, which represent the strong interactions among the participating hadrons, are generally constructed using either effective field theories, such as chiral perturbation theory, or phenomenological Lagrangians, such as the hidden gauge ones. Symmetry arguments play an important role in constructing the potentials and in fixing the unknown parameters. All of the unitarization procedures respect coupled-channel two-body unitarity above thresholds, but may differ in their treatment of off-shell, left-hand cut effects, etc., which, in most cases, induce sub-dominant corrections that are partially accounted for by the undetermined low energy constants. In the present work, we focus on the Bethe-Salpeter equation method based on the so-called on-shell approximation [68–70]. For a discussion of the off-shell effects, see, e.g., Refs. [71, 72] and Ref. [73]. In the latter reference, the off-shell effects are explicitly demonstrated to be small.

The Bethe-Salpeter equation reads, symbolically,

$$T_{ij} = V_{ij} + (VGT)_{ij}, \quad (1)$$

where i, j denote the channel index, V is the kernel potential, T stands for the unitarized amplitude, and G is the two-point one-loop function.

In the study of the $\Lambda_c(2595)$, the relevant kernel potentials V have been explicitly calculated in the framework of chiral [12] and the extended hidden gauge [18] Lagrangians, and in the $SU(6) \times HQSS$ model of Ref. [15]. They differ in the number of included coupled channels and how chiral symmetry and HQSS are taken into account. We refer to Refs. [12, 15, 16, 18] for more details. (A brief revision of the $SU(6) \times HQSS$ model is presented in Subsect. IV C 2).

In addition to the potential, the loop function G in the Bethe-Salpeter equation also plays an important role. It has the following simple form in 4 dimensions:

$$G = i \int \frac{d^4 q}{(2\pi)^4} \frac{2M}{[(P - q)^2 - m^2 + i\epsilon][q^2 - M^2 + i\epsilon]}, \quad (2)$$

with M and m the baryon and meson masses, respectively. This loop function is logarithmically divergent and needs to be properly regularized. Two different methods can be found in the literature: the dimensional regularization scheme and the other in which an ultra-violet hard cut-off is used. In the modified minimal subtraction scheme, the loop function reads

$$\begin{aligned} G_{\overline{MS}}(s, M^2, m^2) = & \frac{2M}{16\pi^2} \left[\frac{m^2 - M^2 + s}{2s} \log \left(\frac{m^2}{M^2} \right) \right. \\ & - \frac{q}{\sqrt{s}} (\log[2q\sqrt{s} + m^2 - M^2 - s] + \log[2q\sqrt{s} - m^2 + M^2 - s]) \\ & - \log[2q\sqrt{s} + m^2 - M^2 + s] - \log[2q\sqrt{s} - m^2 + M^2 + s]) \\ & \left. + \left(\log \left(\frac{M^2}{\mu^2} \right) - 2 \right) \right], \end{aligned} \quad (3)$$

where s is the invariant mass squared of the meson-baryon system. To take into account non-perturbative effects, the constant -2 in the above equation is often replaced by the so-called subtraction constant a , which can be slightly fine-tuned to achieve better agreement with experimental data, in terms of masses and widths of the dynamically generated resonances. An alternative way to fix a is to require that at a certain energy scale, μ_0^2 , the unitarized amplitude reduces to that of the tree level, such as $G(\mu_0^2) = 0$. This has been referred to as the naturalness requirement [14]. In the following, we refer to this regularization method as “DR-naturalness.” It should be noted that this is the method adopted in Refs. [14, 15].

In Ref. [41], a so-called HQS inspired regularization scheme has been suggested, which is manifestly consistent with both the chiral power counting and heavy-quark spin-flavor (SF) symmetry, up to Λ_{QCD}/M_H corrections, where M_H is a generic heavy-hadron mass. In this scheme, referred to as “DR-HQS” in the present work, the loop function G reads:

$$G_{HQS} = G_{\overline{MS}} - \frac{2\mathring{M}}{16\pi^2} \left(\log \left(\frac{\mathring{M}^2}{\mu^2} \right) - 2 \right) + \frac{2m_{\text{sub}}}{16\pi^2} \left(\log \left(\frac{\mathring{M}^2}{\mu^2} \right) + a \right), \quad (4)$$

where m_{sub} is a generic pseudoscalar meson mass, which can take the value of m_π in the u, d flavor case or an average of the pion, kaon, and eta masses in the u, d , and s three flavor case. \mathring{M} is the chiral limit value of the charmed or bottom baryon masses. The apparent renormalization scale dependence originates from that of the dimensional regularization and has little to do with

the ‘‘HQS’’ description (for more details, please see Ref. [12]). Note that in the present case, this scheme is equivalent to the modified minimal subtraction one discussed above. In the numerical calculations, we use $\bar{M} = 2.5349$ GeV, which is the average of the sextet charmed baryon masses, $m_{\text{sub}} = 0.368$ GeV, average of the masses of the pseudoscalar mesons, and $\mu = 1$ GeV. In principle, one could use a different value for \bar{M} in the light baryon sector, but this would be equivalent to the use of different subtraction constants for different channels, which we would like to avoid. Natural values for the subtraction constant, considering the range of baryon masses (i.e., \bar{M}) involved in the present study, lie in the $[-6, -2]$ interval, using $a = -2$ as a reference in the modified minimal subtraction scheme.

The loop function can also be regularized with an ultra-violet hard cutoff, Λ , i.e.,

$$G_{\text{cut}} = \int_0^\Lambda \frac{q^2 dq}{2\pi^2} \frac{E_M + E_m}{2E_M E_m} \frac{2M}{s - (E_M + E_m)^2 + i\epsilon}, \quad (5)$$

with $E_M = \sqrt{q^2 + M^2}$, and $E_m = \sqrt{q^2 + m^2}$. Taking into account the typical size of the hadrons, values around 1 GeV are natural for Λ , although its exact value is in most cases determined from a fit to data.

One of the main objectives of this work is, using potentials constructed in different frameworks [12, 15, 18], to study how the so-called compositeness or the dominance of a certain channel varies with the scheme adopted to regularize the loop function G .

III. THE COMPOSITENESS CONDITION

As mentioned previously, the compositeness analysis proposed by Weinberg in Refs. [20, 21] is only valid for bound states. For resonances, it involves complex numbers and, therefore, a strict probabilistic interpretation is lost. The generalization of the compositeness study for resonances has been put forward by different groups. The weight of a hadron-hadron component in a composite particle is defined as [29]

$$X_i = \text{Re} \tilde{X}_i, \quad (6)$$

with

$$\tilde{X}_i = -g_i^2 \left[\frac{\partial G_i^{II}(s)}{\partial \sqrt{s}} \right]_{s=s_0}, \quad (7)$$

where s_0 is the pole position in the complex s plane, G_i^{II} is the loop function evaluated on the second Riemann sheet (SRS), and g_i is the coupling of the resonance to the channel i , which can

Meson	mass(GeV)	Baryon	mass(GeV)
π	0.13804	N	0.93892
K	0.495645	Λ	1.11568
η	0.54786	Σ	1.19315
ρ	0.77549	Ξ	1.31829
K^*	0.89388	Σ^*	1.38280
ω	0.78265	Ξ^*	1.53180
ϕ	1.01946	Λ_c	2.2865
η'	0.95778	Ξ_c	2.46934
D	1.86723	Σ_c	2.4535
D^*	2.00861	Σ_c^*	2.51807
D_s	1.96830	Ξ'_c	2.57675
D_s^*	2.11210	Ξ_c^*	2.64590

TABLE I. Meson and baryon masses used in the present work.

be obtained as

$$g_i^2 = \lim_{\sqrt{s} \rightarrow \sqrt{s_0}} (\sqrt{s} - \sqrt{s_0}) T_{ii}^{II}, \quad (8)$$

where T_{ii}^{II} is the ii element of the T amplitude on the SRS. For bound states, the quantity \tilde{X}_i is real and it is related to the probability of finding the state in the channel i . For resonances, \tilde{X}_i is still related to the squared wave function of the channel i , in a phase prescription that automatically renders the wave function real for bound states [29], and so it might still be used as a measure of the weight of that hadron-hadron channel in the composition of the resonant state [29, 33].

The deviation of the sum of X_i from unity is related to the energy dependence of the s -wave potential,

$$\sum_i X_i = 1 - Z, \quad (9)$$

where

$$Z = \text{Re}\tilde{Z} = \text{Re} \left(- \sum_{ij} \left[g_i G_i^{II}(\sqrt{s}) \frac{\partial V_{ij}(\sqrt{s})}{\partial \sqrt{s}} G_j^{II}(\sqrt{s}) g_j \right]_{s=s_0} \right). \quad (10)$$

Note that the Eqs. (9) and (10) get support from the sum rule [31–33, 35]

$$-1 = \sum_{ij} g_i g_j \left(\delta_{ij} \left[\frac{\partial G_i^{II}(s)}{\partial \sqrt{s}} \right]_{s=s_0} + \left[G_i^{II}(\sqrt{s}) \frac{\partial V_{ij}(\sqrt{s})}{\partial \sqrt{s}} G_j^{II}(\sqrt{s}) \right]_{s=s_0} \right), \quad (11)$$

which is also satisfied in the case of bound states located in the first Riemann sheet, and guaranties that the imaginary parts of $\sum_i \tilde{X}_i$ and \tilde{Z} must cancel. The field renormalization constant \tilde{Z} itself is well-defined even for resonances, since it corresponds to the residue of the renormalized two-point function [31]. Thus, there is no fundamental problem in calculating \tilde{Z} using Eq. (10), but the probabilistic interpretation of the obtained result is not straightforward. The field renormalization constant \tilde{Z} measures the effect of the elementary contribution as the deviation from unity, and it is in general a complex number. Therefore one should be aware that \tilde{Z} can not directly be interpreted as the “probability” of the elementary component [32]. Conversely, strictly speaking, \tilde{X}_i cannot be interpreted as a probability of finding a two-body component. Nevertheless, because it represents the contribution of the channel wave function to the total normalization, the compositeness \tilde{X}_i will have an important piece of information on the structure of the resonance. In general, however, all \tilde{X}_i and \tilde{Z} can be arbitrary complex numbers constrained by Eq. (11). The probabilistic interpretation of the structure of a resonance from \tilde{X}_i and \tilde{Z} is not possible when the imaginary parts are sizable [33] or when there is a large cancellation among the real parts of $\sum_i \tilde{X}_i$ and \tilde{Z} to meet the sum rule of Eq. (11), but with one of them exceeding the unity. T. Hyodo, following the ideas of T. Berggren [74] in the seventies, has proposed to look at the parameter P , defined as

$$P = |\tilde{Z}| + \left| \sum_i \tilde{X}_i \right| - 1 = |\tilde{Z}| + |1 - \tilde{Z}| - 1 = \left| 1 - \sum_i \tilde{X}_i \right| + \left| \sum_i \tilde{X}_i \right| - 1, \quad (12)$$

and try to give a “probabilistic” interpretation to \tilde{Z} and $\sum_i \tilde{X}_i$ only for those cases where P is much smaller than 1/2 [75].

In the picture advocated in Ref. [29] imaginary parts are neglected. The quantity $1 - Z$ is taken to represent the compositeness of the hadronic state in terms of all the considered channels, and Z is referred to as its elementariness. Within this picture, a non-vanishing Z takes into account that ultimately the model is an effective one. The energy dependent interaction effectively accounts for other possible interaction mechanisms not explicitly included in the s –wave hadron-hadron description. These could be other hadron-hadron interactions, or even genuine hadron components not of the molecular type (hence the appellation elementariness). Thus, a small value of Z indicates that the state is well described by the contributions explicitly considered, namely, s –wave hadron-hadron channels. Conversely, a large value of Z indicates that, for that state, significant pieces of

information are missing in the model, and this information is being included through an effective interaction, to the extent that the experimental hadronic properties are reproduced by the model. However, it is not clear how to interpret Z obtained from the smooth energy dependence of the chiral potential V [30]. In addition, it should be emphasized that, for processes involving short distances, it is the wave function at the origin that matters ($g_i G_i$ for the s wave) [25, 76]³.

On the other hand, in Ref. [38], it was claimed that one can formulate a meaningful compositeness relation with only positive coefficients thanks to a suitable unitary transformation of the S matrix. This in practice amounts to take the absolute value of \tilde{X}_i in Eq. (7) to quantify the probability of finding a specific component in the wave function of a hadron. Notice that the recipe advocated in Ref. [38] is not applicable to all types of poles. In particular the arguments of this reference exclude the case of virtual states or resonant signals which are an admixture between a pole and an enhanced cusp effect by the pole itself. More specifically, the probabilistic interpretation given in [38] to $|\tilde{X}_i|$ is only valid when $\sqrt{\text{Res}_0} > M_{i,\text{th}}$, with $M_{i,\text{th}}$ the corresponding threshold of the i th channel ⁴.

In what follows, we will examine how the number of coupled channels and the particular regularization scheme affect the predicted (calculated) compositeness of the $\Lambda_c(2595)$. For such a purpose, we first fix the number of coupled channels and therefore the kernel potentials, and then compare the resulting compositeness coefficients. The meson and baryon masses employed in the numerical analysis are the same as those used in Ref. [35] and are compiled here in Table I.

According to the PDG, the $\Lambda_c(2595)$ has a mass of 2592.25 ± 0.28 MeV and a width of 2.6 ± 0.6 MeV [1]. Therefore, the only parameter in each of the three regularization schemes discussed in Sect. II is fixed in such a way that the mass of the $\Lambda_c(2595)$ is reproduced. We do not attempt to fix the width because we only consider here two-body coupled channels and work at the isospin symmetric limit, both approximations can have an effect larger on the width than on the mass (see an elaborate discussion in Ref. [37]).

³ For an extensive discussion on this issue, see Ref. [30], where it was concluded that to judge the relevance of each channel one has to study different physical processes.

⁴ In this situation the convergence region of the Laurent series of the S matrix around the pole incorporates some intervals of the physical real axis around the pole mass $M_R (\equiv \sqrt{\text{Res}_0})$, and in these circumstances it follows $|\tilde{X}_i| \leq 1$. Actually, it can be proved that $\sum_i |\tilde{X}_i| \leq 1$, where the sum is only over the channels fulfilling $\sqrt{\text{Res}_0} > M_{i,\text{th}}$ [38]. Thus, the so-called effective elementariness is then defined as $1 - \sum_i |\tilde{X}_i|$, which accounts for the contributions of the heavier channels that do not enter into the sum.

A. Sixteen channels

First, we consider the sixteen channels considered in Refs. [15, 16], making also use of the kernel potentials provided by the $SU(6) \times HQSS$ model derived in these references, and examine the dependence of the compositeness condition on the renormalization/regularization scheme employed to render the loop function ultraviolet-finite.

The $SU(6) \times HQSS$ model used in Refs. [15, 16] is basically a $SU(8)$ SF extension⁵ of the $SU(3)$ chiral WT leading order meson-baryon interaction term, including ground state vector meson and $J^P = 3/2^+$ baryon degrees of freedom. This is actually strictly correct only when coupled channels involving $cc\bar{c}$ components (e.g., doubly charmed baryons and $\bar{D}^{(*)}$ antimesons) are neglected as done in Refs. [15, 16]. These channels are OZI disconnected from those involving just one heavy quark. Note that in the heavy-quark limit, the OZI rule becomes exact because the number of charm quarks and the number of charm antiquarks are separately conserved. (For a more detailed discussion see Ref. [77]). In this framework, there appear two $\Lambda_c(2595)$ states, resemblance of the two $\Lambda(1405)$ resonances found in chiral unitarity approaches, with one of them narrower than the other [15, 16].

To make a reliable comparison, we adjust the only parameter in each of the regularization schemes discussed above to fix the real part of the narrower pole to the $\Lambda_c(2595)$ resonance mass quoted in the PDG [1]. This yields the following parameters, $\alpha = 0.97952$ for the DR-naturalness scheme⁶, $q_{\max} = 0.67898$ GeV for the cutoff scheme, and $a = -3.37865$ for the DR-HQS scheme.

Compositeness results for the $\Lambda_c(2595)$ and its broader partner are shown in Tables II and III, respectively. Among the 16 coupled channels, in general the most relevant ones are $\pi\Sigma_c$, DN and D^*N . In the case of the narrow state (Table II) and for the DR-naturalness scheme, the first of these channels is suppressed, and the dominant components turn out to be DN and D^*N .

For the sibling state of the $\Lambda_c(2595)$, it seems that the $\pi\Sigma_c$ channel plays the dominant role, except in the cutoff scheme, where it appears as a bound state and DN and D^*N channels are more important. For the state that we assign to the $\Lambda_c(2595)$, different regularization schemes yield somehow different results. The D^*N channel plays a leading role in the DR-naturalness scheme of Refs. [15, 16]. In the cutoff scheme, $\pi\Sigma_c$ is the dominant channel, with D^*N the next component in importance. In the DR-HQS scheme, all three mentioned channels seem to be similarly

⁵ This corresponds to treating the eight states of a quark (u, d, s or c with spin up, \uparrow , or down, \downarrow) as equivalent, and leads to the invariance group $SU(8)$. Because $SU(8)$ SF symmetry is strongly broken in nature, mass and weak decay constant breaking effects are taken into account in Refs. [15, 16].

⁶ This is defined for instance in Eq. (17) of Ref. [15]. 11

important, with a large imaginary part for $\tilde{X}_{\pi\Sigma_c}$. On the other hand, when interpreting the compositeness using the prescription of Ref. [38], we find that the weights of $\pi\Sigma_c$ inside the $\Lambda_c(2595)$ are 0.11, 0.71 and 0.97 for the DR-naturalness, cutoff and DR-HQS schemes, respectively. Since the DN , D^*N and other heavier channels do not meet the criterion of Ref. [38], no definite conclusions can be made separately for each of these channels. Besides, $1 - |\tilde{X}_{\pi\Sigma_c}|$ would be the effective elementariness, which get contributions from all of the other heavier channels. Similar conclusions can be also made for the broader state in Table III.

We pay now attention to the uncertainty parameter introduced in Eq. (12). It is significantly smaller than 1/2, which allows for an approximate “probabilistic” interpretation of X_i and Z as advocated in Ref. [29], only in the DR-naturalness and cutoff schemes for the $\Lambda_c(2595)$ and its broader partner, respectively. With larger uncertainties, the DR-HQS scheme for both resonances and the DR-naturalness one for the wider state might also allow for an approximate “probabilistic” interpretation of the results obtained for the different components.

Thus we see the regularization scheme plays a relevant role in the compositeness even with the same number of coupled channels and identical kernel potentials. In other words, the so-called compositeness used in the present way cannot be taken as a model-independent quantity. This is not a surprise, but it reflects the scheme-dependent nature of the field renormalization constant, \tilde{Z} . Similar conclusions have also been reached in Refs. [32, 34].

To finish this subsection, we should note that in the present approach, we have only fitted the mass of the $\Lambda_c(2595)$, while the compositeness coefficients \tilde{X}_i in Eq. (7) depend also on the couplings, which are in turn related to the width. Note that except in the naturalness scheme, the predicted width for the $\Lambda_c(2595)$ turns out to be much larger than its experimental value. A dedicated study including the isospin breaking effects, together with other channels, may provide further insight into the problem (see, e.g., Ref. [37]), which is however beyond the scope of the present study.

B. Two channels

In the unitarized chiral approach of Ref. [12], the $\Lambda_c(2595)$ resonance is dynamically generated from the coupled-channel interaction between only the $\pi\Sigma_c$ and $K\Xi'_c$ meson-baryon pairs. As shown in Table IV, all three regularization schemes considered in this work yield consistent values for the compositeness coefficients, although all with large imaginary parts and leading to values

coupled channels	DR-naturalness	cutoff	DR-HQS
Pole position (MeV)	$2592.25 - i0.16$	$2592.25 - i9.18$	$2592.25 - i3.83$
$\pi\Sigma_c$	$-0.024 + i0.107$	$0.319 + i0.637$	$-0.137 + i0.960$
DN	$0.292 - i0.026$	$0.025 + i0.018$	$0.343 - i0.277$
$\eta\Lambda_c$	$0.009 - i0.001$	$0.004 - i0.001$	$0.040 - i0.042$
D^*N	$0.451 - i0.055$	$0.155 - i0.044$	$0.243 - i0.302$
$K\Xi_c$	$0.001 - i0.000$	$0.000 - i0.000$	$0.001 - i0.001$
$\omega\Lambda_c$	$0.001 - i0.000$	$-0.000 - i0.001$	$0.014 - i0.012$
$K\Xi'_c$	$0.000 + i0.000$	$0.000 - i0.001$	$0.002 - i0.001$
$D_s\Lambda$	$0.026 - i0.003$	$0.004 - i0.000$	$0.018 - i0.019$
$D_s^*\Lambda$	$0.057 - i0.006$	$0.008 - i0.001$	$0.051 - i0.054$
$\rho\Sigma_c$	$0.005 - i0.000$	$-0.000 - i0.002$	$0.007 - i0.004$
$\eta'\Lambda_c$	$0.018 - i0.002$	$0.003 - i0.000$	$0.018 - i0.019$
$\rho\Sigma_c^*$	$0.006 - i0.001$	$0.003 - i0.002$	$0.006 - i0.008$
$\phi\Lambda_c$	$-0.000 - i0.000$	$-0.000 - i0.000$	$0.000 - i0.000$
$K^*\Xi_c$	$0.000 + i0.000$	$0.000 - i0.000$	$0.001 - i0.001$
$K^*\Xi'_c$	$0.000 - i0.000$	$-0.000 - i0.000$	$-0.000 - i0.000$
$K^*\Xi_c^*$	$0.000 - i0.000$	$0.000 - i0.000$	$0.000 - i0.000$
$\sum_i \tilde{X}_i$	$0.843 + i0.012$	$0.521 + i0.602$	$0.607 + i0.219$
P [Eq. (12)]	0.001	0.565	0.095

TABLE II. Compositeness \tilde{X}_i of each of the 16 coupled channels for the narrow state corresponding to the $\Lambda_c(2595)$. The potentials V are those of the $SU(6)\times HQSS$ model of Refs. [15, 16]. The finite (renormalized) meson-baryon loop function is fitted to the $\Lambda_c(2595)$ mass. This leads to the following parameters: $\alpha = 0.97952$, $q_{\max} = 0.67898$ GeV, $a = -3.37865$ in the DR-naturalness, cutoff and the DR-HQS schemes, respectively. The real parts of the \tilde{X}_i coefficients, calculated within the DR-naturalness renormalization scheme, were already given in Table IV of Ref. [35]. According to Ref. [38], it is only meaningful to give a probabilistic interpretation to $|\tilde{X}_{\pi\Sigma_c}|$.

coupled channels	DR-naturalness	cutoff	DR-HQS
Pole position (MeV)	$2606.7 - i32.4$	2572.2	$2627.9 - i37.4$
$\pi\Sigma_c$	$0.307 + i0.429$	0.041	$0.494 + i0.109$
DN	$0.005 - i0.044$	0.254	$-0.115 + i0.001$
$\eta\Lambda_c$	$0.000 + i0.000$	0.009	$0.014 + i0.024$
D^*N	$0.048 + i0.024$	0.278	$0.322 + i0.172$
$K\Xi_c$	$-0.000 + i0.000$	0.001	$-0.000 + i0.001$
$\omega\Lambda_c$	$0.001 - i0.006$	0.001	$-0.005 + i0.002$
$K\Xi'_c$	$0.001 - i0.005$	0.000	$-0.001 - i0.004$
$D_s\Lambda$	$-0.000 + i0.001$	0.012	$0.006 + i0.011$
$D_s^*\Lambda$	$0.001 + i0.002$	0.021	$0.016 + i0.029$
$\rho\Sigma_c$	$0.013 - i0.027$	0.002	$0.000 - i0.012$
$\eta'\Lambda_c$	$0.000 + i0.001$	0.007	$0.007 + i0.011$
$\rho\Sigma_c^*$	$0.007 - i0.006$	0.002	$0.015 + i0.001$
$\phi\Lambda_c$	$-0.000 - i0.000$	-0.000	$0.000 + i0.000$
$K^*\Xi_c$	$0.002 - i0.004$	0.000	$0.000 - i0.002$
$K^*\Xi'_c$	$0.000 - i0.002$	0.000	$0.001 - i0.001$
$K^*\Xi_c^*$	$0.000 - i0.001$	0.000	$-0.000 - i0.001$
$\sum_i \tilde{X}_i$	$0.388 + i0.363$	0.616	$0.755 + i0.339$
P [Eq. (12)]	0.243	0.000	0.246

TABLE III. Same as in Table II, but for the broader sibling of the $\Lambda_c(2595)$ resonance.

of the uncertainty parameter P well above 1/2. Moreover the values for $\tilde{X}_{\pi\Sigma_c}$ listed in Table IV significantly differ from those obtained in the 16 channel case of Table II.

The cutoff and the DR-HQS subtraction-constant needed to fit the $\Lambda_c(2495)$ mass turn out to be rather natural (see the discussion in Sect. II), while the α parameter in the DR-naturalness scheme

coupled channels	DR-naturalness	cutoff	DR-HQS
Pole position (MeV)	$2592.25 - i12.7$	$2592.25 - i15.6$	$2592.25 - i13.5$
$\pi\Sigma_c$	$0.215 + i0.731$	$0.196 + i0.770$	$0.225 + i0.720$
$K\Xi'_c$	$0.003 - i0.006$	$0.001 - i0.002$	$0.003 - i0.007$
$\sum_i \tilde{X}_i$	$0.218 + i0.725$	$0.196 + i0.768$	$0.228 + i0.713$
P [Eq. (12)]	0.823	0.904	0.799

TABLE IV. Compositeness \tilde{X}_i for the $\Lambda_c(2595)$ obtained when only the $\pi\Sigma_c$ and $K\Xi'_c$ channels are considered, as in the chiral approach of Ref. [12]. For all renormalization schemes, the coupled-channel matrix potential V is taken from this reference (note the approaches of Refs. [15, 16, 18] provide the same interaction, since it is fixed by SU(3) chiral symmetry). The finite (renormalized) meson-baryon loop function is fitted to the $\Lambda_c(2595)$ mass. This leads to the following parameters: $\alpha = 0.8268$, $q_{\text{max}} = 0.7969$ GeV, and $a = -5.3768$ in the DR-naturalness, cutoff and DR-HQS schemes, respectively.

deviates appreciably from 1.

Similar conclusions are drawn in the single channel case, $\pi\Sigma_c$, independently of the value used for the decay constant.

C. Three channels

In the local hidden gauge approach of Ref. [18], three channels are considered, namely $\pi\Sigma_c$, DN , and $\eta\Lambda_c$. Taking the kernel potentials from Ref. [18], we calculate the compositeness coefficients \tilde{X}_i using the three regularization schemes introduced in the previous subsections. Results are shown in Table V. We can see that in the DR-naturalness scheme, the DN channel dominates, while in the DR-HQS method, the $\pi\Sigma_c$ component is the most significant. The renormalization method has an important impact on the compositeness coefficients, despite all renormalization constants have been adjusted to reproduce the mass of the Λ_c resonance.

We would like to make a further remark here. In the DR-naturalness scheme, the consideration of the DN channel has led to a value for α quite close to 1, and an uncertainty parameter P [Eq. (12)] very small, enabling for a “probabilistic” interpretation. Note that, however, the

coupled channels	DR-naturalness	cutoff	DR-HQS
Pole position (MeV)	$2592.25 - i0.86$	$2592.25 - i11.4$	$2592.25 - i12.1$
$\pi\Sigma_c$	$-0.060 + i0.483$	$0.057 + i1.002$	$0.212 + i0.729$
DN	$0.815 - i0.390$	$0.136 - i0.355$	$0.001 - i0.001$
$\eta\Lambda_c$	$0.017 - i0.008$	$0.002 - i0.006$	$0.001 - i0.001$
$\sum_i \tilde{X}_i$	$0.772 + i0.085$	$0.195 + i0.641$	$0.214 + i0.727$
P [Eq. (12)]	0.020	0.699	0.829

TABLE V. Compositeness \tilde{X}_i for the $\Lambda_c(2595)$ resonances obtained considering three, $\pi\Sigma_c$, DN and $\eta\Lambda_c$, channels as in the extended hidden gauge approach of Ref. [18]. For all renormalization schemes, the coupled-channel matrix potential V is taken from this reference. The finite (renormalized) meson-baryon loop function is fitted to the $\Lambda_c(2595)$ mass. This leads to the following parameters: $\alpha = 0.96048$, $q_{\max} = 0.67535$ GeV, and $a = -5.6365$ in the DR-naturalness, cutoff and DR-HQS schemes, respectively.

P -values obtained in the other two renormalization schemes are larger than 1/2, since in both cases the imaginary parts of $\sum_i \tilde{X}_i$ are much larger than the real ones.

IV. LARGE N_c EVOLUTION

The N_c counting rules for ordinary qqq baryons lead to scaling laws $\Gamma_R \sim \mathcal{O}(1)$, $M_R \sim \mathcal{O}(N_c)$ and $\triangle E \equiv M_R - M_B - m \sim \mathcal{O}(1)$, with $M_B(m)$ the ground-state baryon (meson) mass, for the resonance decay width, mass and excitation energy, respectively [60, 66, 67]. For an ordinary $q\bar{q}$ state, its mass, width and decay constant scale as $\mathcal{O}(1)$, $\mathcal{O}(1/N_c)$ and $\mathcal{O}(\sqrt{N_c})$, respectively. For dynamically generated states, the N_c -evolution can deviate strongly from such a scenario [45, 47, 50, 53, 54]. Compared to the dynamically generated mesons, a study of dynamically generated baryonic states is complicated because baryon flavor representations change with N_c , when the number of flavors is larger than two [78–80]. Such corrections have been taken into account in the SU(3) chiral study of the $\Lambda(1405)$ in Refs. [57, 58], as well as in the study of negative parity s -wave resonances carried out in [55, 56], where a SU($2N_F$) SF extension of the chiral SU(3) WT interaction for an arbitrary number of flavors and colors is derived. In the present exploratory work

on the $\Lambda_c(2595)$, we will present $N_c > 3$ results for the chiral two coupled-channel scenario [12], and only in the strict $N_c \rightarrow \infty$ limit, in the case of the $SU(6) \times \text{HQSS}$ model [15, 16].

To obtain the large N_c evolution of the dynamically generated states in unitarized approaches, one needs to know how the masses of the interacting hadrons, the two body loop function, and the interactions evolve as a function of N_c . The latter evolution is partially a consequence of the change of the flavor representation of the baryons. In what follows, we examine the N_c dependence of all these inputs.

A. Baryon and meson masses

Ground-state heavy flavor baryon masses in the $1/m_Q$ and $1/N_c$ expansions have been studied in Refs. [81–83]. Up to leading order in $1/N_c$, one has

$$M_i = m_Q + M_0 \frac{N_c}{3} + \delta_i, \quad (13)$$

where m_Q is the N_c independent heavy quark mass, $M_0/3$ the contribution of the light u , d , s quarks, and δ_i the flavor $SU(3)$ breaking contributions. For the present study, we take $m_Q = m_c = 1.275$ GeV, $M_0 \sim 0.9$ GeV, and δ_i is chosen such that M_i equals to its physical value for $N_c = 3$. The pseudoscalar meson masses scale as $\mathcal{O}(1)$ and are taken as constants, while the pseudoscalar decay constant scales as $\mathcal{O}(\sqrt{N_c})$, namely,

$$f(N_c) = f_0 \sqrt{\frac{N_c}{3}}, \quad f_0 = f(N_c = 3). \quad (14)$$

B. Loop function

As already mentioned, the meson-baryon loop function in Eq. (2) is logarithmically divergent and should be regularized. For that purpose in this work we have used either the dimensional regularization method or have included a momentum cutoff to render the ultraviolet contributions finite. This latter scheme, Eq. (5), is particularly useful, because its extension to arbitrary N_c might be more transparent.

For $N_c = 3$, the cutoff takes values of the order of 1 GeV. Although the N_c behavior of the cutoff is not known from QCD, it is, however, clear that within the chiral approach used in Ref. [12], it cannot grow faster than the cutoff of the effective theory itself, which is of the order of the scale

of symmetry breaking $\Lambda_\chi \sim 4\pi f$. Otherwise, we would have the absurd situation that we can extend the validity of the loop integral beyond the applicability of the theory. Therefore, a natural integral cutoff, as is the case here, could scale as $\sqrt{N_c}$, but not faster [49]. We will also consider the possibility that the cutoff may scale slower than $\sqrt{N_c}$, since it would be $\mathcal{O}(1)$, if it were determined by the existence of heavier qqq states, which cannot be generated from low-energy baryon-meson dynamics, and therefore have been integrated out. We will present results for both scenarios, which yield consistent conclusions, as it will be shown below.

In the dimensional regularization scheme, the mayor problem arises from the unknown N_c dependence of the subtraction constant, a . However, in the DR-naturalness scheme, it is given in terms of the meson and baryon masses [14, 15], which in turn fix the full dependence of the loop function on N_c . This scheme was employed in Ref. [55] to study the properties of the negative parity s -wave resonances in the large N_c limit, starting from a SU(6) spin–light flavor extension of the chiral WT interaction for $N_c = 3$. Indeed, some expressions given in that reference were more general, and can be applied to the SU($2N_F$) group symmetry for an arbitrary N_c . We will take advantage of these findings and will use the framework set up in Refs. [55, 56] to discuss the strict $N_c \rightarrow \infty$ limit of the SU(6) \times HQSS model used in Refs. [15, 16].

C. N_c dependence of the meson-baryon interaction

1. $K\Xi'_c - \pi\Sigma_c$ chiral interaction

In the unitary approach of Ref. [12], the $\Lambda_c(2595)$ resonance is dynamically generated from the chiral interaction between the pseudoscalar octet of Goldstone bosons and the sextet (Σ_c, Ξ'_c) of charmed baryons⁷. In the strangenessless ($S = 0$) isoscalar ($I = 0$) sector the interaction reads [12]

$$V^{I=0,S=0}(s) = \frac{C^{I=0,S=0}}{4f^2}(E_m + E'_m), \quad (15)$$

⁷ In the heavy quark limit, the spin-parity of the light degrees of freedom in these baryons is 1^+ .

with E_m and E'_m the center of mass energies of the initial and final mesons, respectively and the coupled-channel matrix is given by

$$C^{I=0,S=0} = \begin{pmatrix} K\Xi'_c & \pi\Sigma_c \\ -2 & -\sqrt{3} \\ -\sqrt{3} & -4 \end{pmatrix} \begin{pmatrix} K\Xi'_c \\ \pi\Sigma_c \end{pmatrix}, \quad (16)$$

In the SU(3) group theory language we have:

$$8 \otimes 6 = \bar{3} \oplus 6 \oplus \bar{15} \oplus \bar{24}. \quad (17)$$

Although the decomposition involves four SU(3) irreducible representations, only the $\bar{3}$ and $\bar{15}$ appear in the $I = 0, S = 0$ sector. Thus, the coupled-channel matrix $C^{I=0,S=0}$ becomes diagonal in the $\{|\bar{3}; I = 0, S = 0\rangle, |\bar{15}; I = 0, S = 0\rangle\}$ SU(3) basis. The meson-baryon and the SU(3) bases are related by means of an orthogonal matrix U obtained from the appropriate SU(3) Clebsch-Gordan coefficients [84]

$$\left(|\bar{3}\rangle, |\bar{15}\rangle\right) = \left(|K\Xi'_c\rangle, |\pi\Sigma_c\rangle\right) \times U, \quad U = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}. \quad (18)$$

In the SU(3) basis, the interaction of Eq. (15) reads

$$C_{\text{SU}(3)}^{I=0,S=0} = U^\dagger C^{I=0,S=0} U = \begin{pmatrix} -5 & 0 \\ 0 & -1 \end{pmatrix}. \quad (19)$$

While in the meson sector, the flavor representation remains the same with the increase of N_c , the situation in the baryon sector is more complicated because of the nontrivial variation of the flavor representation of the baryons with N_c , when the number of flavors is larger than 2 [78, 79]. We use the notation $[p, q]$ for an irreducible representation of SU(3), whose corresponding Young tableau has $p + q$ and q boxes in the first and second rows, respectively. To extend the irreducible flavor representation from $N_c = 3$ to arbitrary N_c , we adopt the prescription ⁸

$$[p, q] \rightarrow [p, q + \frac{N_c - 3}{2}], \quad (20)$$

⁸ There are two other alternative ways to perform the extension. The one used in the present work, referred to as the standard one in Ref. [79], has the advantage of keeping the spin, isospin, strangeness and charm quantum numbers of the original representation at $N_c = 3$, while the baryons have different charge and hypercharge from those at $N_c = 3$.

For arbitrary N_c , the 6 , $\bar{3}$, and $\bar{15}$ irreducible representations become (we use the notation that an N_c -representation “ R ” reduces to R at $N_c = 3$ [78–80]),

$$\begin{aligned} \text{“}6\text{”} &= [2, \frac{N_c - 3}{2}], \\ \text{“}\bar{3}\text{”} &= [0, \frac{N_c - 1}{2}], \\ \text{“}\bar{15}\text{”} &= [1, \frac{N_c + 1}{2}]. \end{aligned} \quad (21)$$

From group theory the SU(3) basis coupling strengths (eigenvalues) for arbitrary N_c turn out to be (see Table III of Ref. [85]):

$$C_{\text{SU}(3)}^{I=0, S=0}(N_c) = \begin{pmatrix} -5 & 0 \\ 0 & -\frac{5-N_c}{2} \end{pmatrix}, \quad (22)$$

which reduces to Eq. (19) at $N_c = 3$. Note that the “ $\bar{15}$ ” eigenvalue becomes repulsive for $N_c > 5$, while the interaction in the “ $\bar{3}$ ” subspace is always attractive and independent of N_c , besides the scaling of the decay constant and masses in Eq. (15).

The transformation matrix U will now depend on N_c as well. It can be obtained from the appropriate N_c dependent SU(3) Clebsch-Gordan coefficients. Using the recursion relations of Ref. [86] or the results of Ref. [87], one can easily obtain the explicit form of $U(N_c)$ for the decomposition $8 \otimes \text{“}6\text{”} = \text{“}\bar{3}\text{”} \oplus \text{“}\bar{15}\text{”} \oplus \text{“}6\text{”} \oplus \text{“}\bar{24}\text{”}$.

Following the usual convention, the SU(3) Clebsch-Gordan (CG) coefficients can be expressed as the products of isoscalar factors and ordinary SU(2) CGCs.

$$\left(\begin{array}{cc|c} R_1 & R_2 & R_\gamma \\ I_1, I_{1z}, Y_1 & I_2, I_{2z}, Y_2 & I, I_z, Y \end{array} \right) = \left(\begin{array}{cc|c} R_1 & R_2 & R_\gamma \\ I_1, Y_1 & I_2, Y_2 & I, Y \end{array} \right) \left(\begin{array}{cc|c} I_1 & I_2 & I \\ I_{1z} & I_{2z} & I_z \end{array} \right) \quad (23)$$

where the label R indicates the SU(3) representation, which can be denoted using the usual weight diagram notation (λ, μ) , and γ labels degenerate representations occurring in a given product.

With the formula given in Table 4 of Ref. [87], the transformation matrix U can be obtained straightforwardly. The first element, for instance, should be

$$U_{11} = \sqrt{\frac{(p+1)(\lambda-1-p)q(\lambda+\mu+1-q)(\lambda+\mu+2-q)}{\lambda(\lambda+1)(\mu+1)(\lambda+\mu+2)(\mu+p-q+2)}} \quad (24)$$

with

$$p = \frac{Y}{2} + I + \frac{\lambda' - \mu'}{3}, \quad q = \frac{Y}{2} - I + \frac{\lambda' + 2\mu'}{3}, \quad (25)$$

and Y is related with the ϵ of Ref. [87] via $Y = -\epsilon/3$. For the present case, $Y = (N_c - 1)/3$ and $I = 0$. (λ', μ') refer to the representation labeled by “ $\bar{3}$ ” and “ $\bar{15}$ ” and their values are given in Eq. (21). Keeping in mind that the formula above is used to calculate the isoscalar factors of “ 6 ” $\otimes 8$, an extra step is needed to obtain the U matrix for $8 \otimes$ “ 6 ”. Finally, the U matrix can be written as

$$U(N_c) = \begin{pmatrix} -\sqrt{\frac{2}{5+N_c}} & -\sqrt{\frac{3+N_c}{5+N_c}} \\ -\sqrt{\frac{3+N_c}{5+N_c}} & \sqrt{\frac{2}{5+N_c}} \end{pmatrix}. \quad (26)$$

With all these ingredients, we finally obtain the $K\Xi'_c - \pi\Sigma_c$ coupled-channel interaction for an arbitrary number of colors N_c

$$C^{I=0, S=0}(N_c) = U(N_c) \left[C_{\text{SU}(3)}^{I=0, S=0}(N_c) \right] U^\dagger(N_c) = \begin{pmatrix} \frac{N_c-7}{2} & -\sqrt{\frac{N_c+3}{2}} \\ -\sqrt{\frac{N_c+3}{2}} & -4 \end{pmatrix}. \quad (27)$$

It is interesting to note that the $\pi\Sigma_c \rightarrow \pi\Sigma_c$ interaction is attractive and does not change with N_c , while the $K\Xi'_c$ self-interaction, which is attractive at $N_c = 3$, becomes repulsive for $N_c > 7$. On the other hand, the strength of the off-diagonal transition increases with N_c .

2. $SU(6) \times \text{HQSS}$

To better understand the $N_c \gg 1$ limit of the $SU(6) \times \text{HQSS}$ model, we need to give some further details on its main features. The 16 coupled-channel model implemented in Refs. [15, 16] has its origin in the compatibility between SF and chiral symmetries, which implies that the WT interaction can be extended to enjoy SF invariance [$SU(2N_F)$]. Actually this can be done in a unique way, as it was demonstrated in [88]. The model respects SF symmetry in the light sector and HQSS in the heavy one, and it reduces to $SU(3)$ WT in the light sector respecting chiral symmetry. HQSS connects vector and pseudoscalar mesons containing charmed quarks. On the other hand, chiral symmetry fixes the lowest-order interaction between Goldstone bosons and other hadrons in a model-independent way; this is the WT interaction.

As required by SF symmetry, the model of Refs. [15, 16] incorporates ground state vector meson and $J^P = 3/2^+$ baryon degrees of freedom, in addition to the ground state pseudoscalar mesons and $J^P = 1/2^+$ baryons. In the large N_c limit, SF becomes exact for the baryon sector [89]. As for mesons, the lowest-lying states can also be classified quite naturally according to

SF multiplets. Though for charmed mesons SF symmetry reduces to HQSS, the symmetry works worse for the light meson spectrum.

SF guarantees HQSS except when there are simultaneously c quarks and \bar{c} antiquarks. This is because SF implies invariance under equal rotations for c and \bar{c} , but HQSS also requires invariance when the two spin rotations are different. Thus, SF does not guaranty HQSS in sectors with hidden charm, regardless of whether they have net charm or not. As mentioned in Subsect. III A, in the study of the $C(\text{charm}) = 1$ sector carried out in Refs. [15, 16] the WT SU(8) interaction kernel was modified, besides using physical masses and weak decay constants, by neglecting the hidden charm $cc\bar{c}$ channels to accomplish HQSS. The model was quite successful and it naturally led to the dynamical generation of the $J^P = 1/2^- \Lambda_c(2595)$ and $J^P = 3/2^- \Lambda_c(2625)$ resonances, among others. Moreover, it could be used to classify the predicted states in $SU(6) \times \text{HQSS}$ multiplets [16]. Its extension to the bottom sector [17] easily accommodated two narrow baryon resonances with beauty recently observed by the LHCb Collaboration [90], that should be intimately related to the charmed $\Lambda_c(2595)$ and $\Lambda_c(2625)$ states.

We do not have the mathematical tools to extend the $SU(6) \times \text{HQSS}$ model to an arbitrary number of colors, and this is beyond the scope of this work. However, some results for the $SU(2N_F)$ WT interaction and an arbitrary number of colors were obtained in Refs. [55, 56]. The $SU(2N_F)$ WT interaction for each $JISC$ sector⁹ reads as that in Eq. (15), but replacing the coupled-channel matrix there by the appropriate one, C^{JISC} , in each sector. Thus for instance, in the $\Lambda_c(2595)$ sector, the dimension of the coupled-channel space is 21: the sixteen channels enumerated in Tables II and III plus the hidden charm channels, $\Lambda_c \eta_c$, $\Lambda_c J/\Psi$, $\Xi_{cc} \bar{D}$, $\Xi_{cc} \bar{D}^*$ and $\Xi_{cc}^* \bar{D}^*$. These latter five channels were neglected in Refs. [15, 16] to restore HQSS symmetry. As discussed in Refs. [15, 16], the SU(8) group reduction¹⁰

$$63 \otimes 120 = 120 \oplus 168 \oplus 2520 \oplus 4752, \quad (28)$$

shows that in the SU(8) basis, there exist only four eigenvalues, associated to each of the irreducible representations that appear on the right hand side of Eq. (28). (Note that the SU(4) **15**-plet of pseudoscalar ($D_s, D, K, \pi, \eta, \eta_c, \bar{K}, \bar{D}, \bar{D}_s$) and the **16**-plet of vector ($D_s^*, D^*, K^*, \rho, \omega, J/\Psi, \bar{K}^*$,

⁹ Here J stands for the total spin of the meson-baryon pair, and for $N_F > 4$, additional flavor quantum numbers would need to be specified.

¹⁰ For any N_F , there always appears four irreducible representations in the group reduction of Eq. (28). Obviously, the dimensions of them, as well as those of the representations where ground state baryons and mesons are included depend on N_F . These latter ones are always the adjoint and the three quark fully symmetric representations, respectively.

D	λ_D	“D”	$\lambda_{\text{“D”}}$
$[N_F = 4, N_c = 3]$			
120	-16	$d(N_F, N_c) = \frac{(2N_F+N_c-1)!}{(2N_F-1)!N_c!}$	$-4N_F$
168	-22	$\frac{(2N_F-1)(N_c-1)}{(2N_F+N_c-1)} \times d(N_F, N_c)$	$-2(N_c + 2N_F)$
2520	6	$\frac{(2N_F-1)(2N_F+N_c+1)}{(N_c+1)} \times d(N_F, N_c)$	$2N_c$
4752	-2	$\frac{2N_F N_c (2N_F+N_c)(2N_F-2)}{(N_c+1)(2N_F+N_c-1)} \times d(N_F, N_c)$	-2

TABLE VI. Dimensions (D) and WT eigenvalues (λ_D) associated to the $SU(2N_F)$ irreducible representations that appear in the group decomposition that generalizes Eq. (28) [$N_F = 4$ and $N_c = 3$], for arbitrary number of flavors and colors. It corresponds to the reduction of the product of the $SU(8)$ adjoint (mesons) and the N_c -quark fully symmetric (baryons) representations (see Eq. (27) of Ref. [55]). Note also a misprint in the expression given in Ref. [56] for the dimension of the “2520” representation.

$\bar{D}^*, \bar{D}_s^*, \phi$) mesons are placed in the **63** representation. The lowest-lying baryons are assigned to the **120** of $SU(8)$. This is appropriate because in the light sector it can accommodate an octet of spin-1/2 baryons and a decuplet of spin-3/2 baryons which are precisely the $SU(3)$ -spin combinations of the low-lying baryon states (N, Σ, Λ, Ξ and $\Delta, \Sigma^*, \Xi^*, \Omega$). The remaining states in the $20_{J=1/2}$ and $20'_{J=3/2}$ are completed with the charmed baryons: $\Lambda_c, \Sigma_c, \Xi_c, \Xi'_c, \Omega_c, \Xi_{cc}, \Omega_{cc}$ and $\Sigma_c^*, \Xi_c^*, \Omega_c^*, \Omega_{cc}^*, \Xi_{cc}^*, \Omega_{ccc}$, respectively.

The eigenvalues associated to the decomposition of Eq. (28) were calculated in [55, 56], for an arbitrary number of colors and not only for $SU(8)$, but for $SU(2N_F)$ in general, and are compiled here in Table VI. Independently of N_c , in the group reduction that generalizes Eq. (28), there only appear four irreducible representations [55]. For four flavors, the $\Lambda_c(2595)$ state belongs to the attractive **168** representation [16], whose attraction linearly grows with N_c . In this subspace, and keeping in mind the $1/f^2$ factor, the WT is always attractive and it scales as $\mathcal{O}(1)$, in the large N_c limit. However in the subspaces associated to the other three representations, the WT interaction is either repulsive or suppressed, $\mathcal{O}(1/N_c)$, when $N_c \gg 3$.

In the $SU(8)$ basis, the coupled-channel interaction matrix $C_{SU(2N_F)}^{JISC}$ is diagonal, however we do not know, for arbitrary N_c , the orthogonal matrix $U_{SU(8)}(N_c)$ that would transform this diagonal matrix into C^{JISC} , the matrix expressed in the meson-baryon basis. It would be obtained from

the appropriate N_c dependent SU(8) Clebsch-Gordan coefficients¹¹. This prevents us to obtain the evolution of the $\Lambda_c(2595)$ pole for moderate values of $N_c > 3$, but however as we will discuss in the next subsection, we will be able to address its behavior for $N_c \gg 3$, where we could consider the loop function diagonal in the meson-baryon basis, as it was done in Ref. [55].

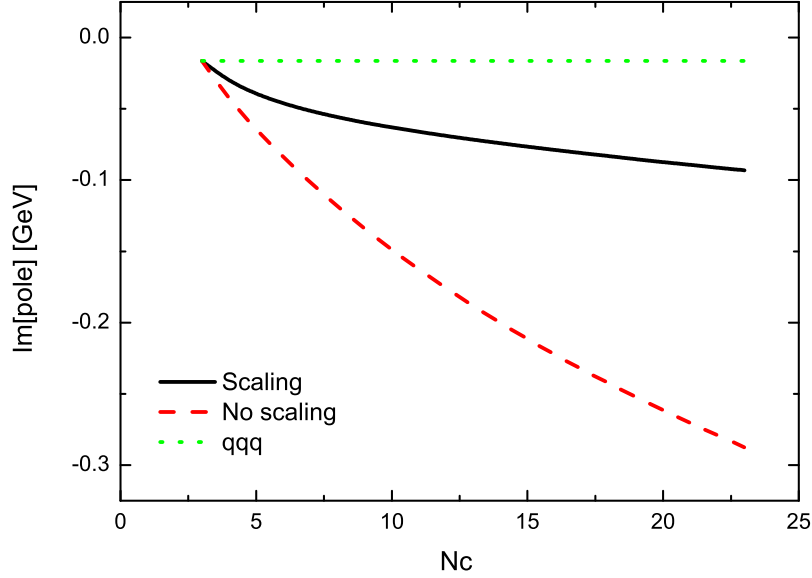


FIG. 1. Imaginary part of the $\Lambda_c(2595)$ pole position as a function of the number of colors. Results have been obtained using the $N_c > 3$ extended coupled-channel $K\Xi'_c - \pi\Sigma_c$ chiral interaction constructed out Eqs. (15) and (27), and employing an ultraviolet-cutoff to render the loop function finite. Curves denoted as “Scaling” and “No scaling” stand for the results obtained with different N_c scaling laws for the cutoff, either $\mathcal{O}(\sqrt{N_c/3})$ or $\mathcal{O}(1)$, respectively.

D. $\Lambda_c(2595)$ mass and width for large N_c

From the findings of the previous subsections it is straightforward to study the N_c dependence of the $\Lambda_c(2595)$ mass and width, when it is dynamically generated from the coupled-channel $K\Xi'_c - \pi\Sigma_c$ chiral interaction. We use an ultraviolet cutoff to renormalize the loop function, and

¹¹ These coefficients can be found in Ref. [84] only for the $N_c = 3$ case.

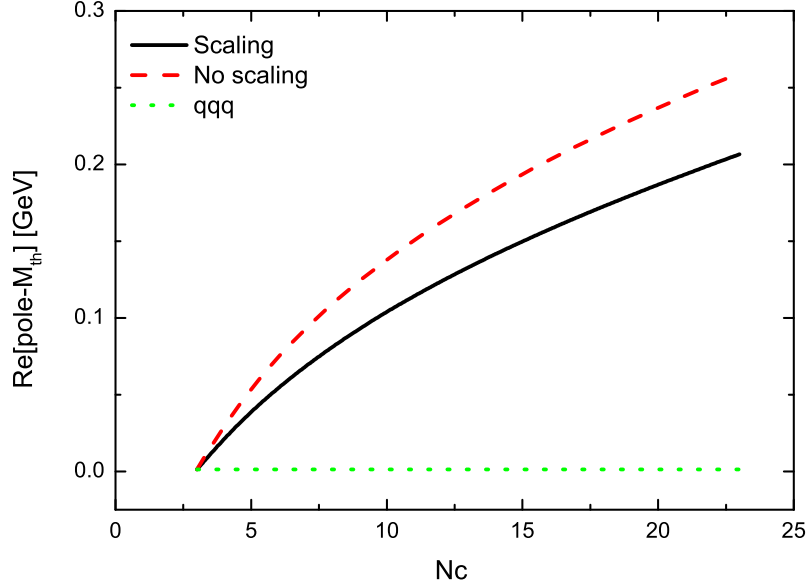


FIG. 2. The same as in Fig. 1, but for the real part of the $\Lambda_c(2595)$ pole position, with respect to the $\pi\Sigma_c$ threshold.

examine two different N_c scaling laws, $\mathcal{O}(\sqrt{N_c/3})$ or $\mathcal{O}(1)$, for this parameter of the effective theory.

Results are displayed in Figs. 1 and 2, where imaginary and real parts of the $\Lambda_c(2595)$ pole position, together with the expected behavior of a conventional qqq baryon, are shown as a function of N_c . We pay attention to moderately large number of colors, up to $N_c = 25$. For both scaling laws of the cutoff, we find that both mass and width of the resonance grow with N_c , more rapidly when the cutoff is taken as constant. Indeed, the resonance tends to disappear since it becomes quite wide (width of hundreds of MeV) and located also hundreds of MeV above the $\pi\Sigma_c$ threshold. This behavior significantly deviates from that expected for a genuine qqq state. Thus, the N_c evolution supports the conjecture that the meson-baryon component in the wave-function of the $\Lambda_c(2595)$ plays a relevant role.

The above analysis is not consistent with the spin symmetry in the baryon sector, though it only becomes exact in the large N_c limit [89], and thus one should be cautious about the consequences extracted in such a scheme. This has motivated us to study the N_c -evolution of the $\Lambda_c(2595)$ -pole position from a different perspective, implementing exact SU(8) SF symmetry.

As discussed in Subsect. IV C 2, we cannot accurately study moderate values of $N_c > 3$ in this context, because we do not know the orthogonal matrix $U_{\text{SU}(8)}(N_c)$, which implements the change of basis between the $\text{SU}(8)$ one and that constructed out of the meson-baryon pairs. Yet, even if we knew such rotation, the obtained results for moderate N_c values would not be physical because SF symmetry does not guaranty HQSS in this intermediate regime. However, for sufficiently large values of N_c , all meson masses become negligible as compared to those of the baryons, all of which in turn, to a good approximation, have a common mass \hat{M} , proportional to N_c , as inferred from Eq. (13),

$$\hat{M} = M_0 \frac{N_c}{3} + \mathcal{O}(1/N_c) \quad (29)$$

In the charm sector $C = 1$, there still appear only two types of configurations involving either only a quark c or an additional $c\bar{c}$ pair, since there is always at most only one charm quark. Since the heavy quark mass is not much larger than the typical scale associated to the cloud of light degrees of freedom, and as N_c increases, the $\text{SU}(8)$ SF symmetry should become more and more accurate. Thus, the pole positions could be obtained in each JIS sector and $C = 1$ from (for simplicity, we drop out the label $JISC$)

$$\det [I - V(s)G^{II}(s)]_{N_c \gg 3} = 0 \quad (30)$$

with $G^{II}(s)$, the matrix loop function calculated in the SRS. In the DR-naturalness renormalization scheme, $G^{II}(s)$ becomes diagonal in the meson-baryon coupled-channel basis as it does the factor $(E_m + E'_m)/f^2 \sim 2(\sqrt{s} - \hat{M})/f^2$ in the definition of the potential¹² in Eq. (15). Under these circumstances, the resonance position equation becomes

$$\begin{aligned} \det [I - V(s)G^{II}(s)]_{N_c \gg 3} &= \det \left[I - \frac{\sqrt{s} - \hat{M}}{2f^2} G^{II}(s) U_{\text{SU}(8)}(N_c) C_{\text{SU}(8)} U_{\text{SU}(8)}^\dagger(N_c) \right]_{N_c \gg 3} \\ &= \det \left[U_{\text{SU}(8)} U_{\text{SU}(8)}^\dagger - \frac{\sqrt{s} - \hat{M}}{2f^2} G^{II}(s) U_{\text{SU}(8)} C_{\text{SU}(8)} U_{\text{SU}(8)}^\dagger \right]_{N_c \gg 3} \\ &= \left[\frac{\sqrt{s} - \hat{M}}{2f^2} G^{II}(s) \right]^n \det [\beta(s) - C_{\text{SU}(8)}]_{N_c \gg 3} = 0 \end{aligned} \quad (31)$$

with $\beta(s) = 2f^2 / [(\sqrt{s} - \hat{M})G^{II}(s)]$ and $C_{\text{SU}(8)}$ a diagonal matrix constructed out of the four eigenvalues, $\lambda_{\text{“D”}}$, given in Table VI. Besides, n is the dimension of the space ($n = 21$ in the $\Lambda_c(2595)$ sector). We see how in the large N_c limit, we can determine the pole position independently of the orthogonal transformation $U_{\text{SU}(8)}(N_c)$. Thus, the pole positions are determined

¹² We are also neglecting SF symmetry breaking effects in the weak decay constants.

by

$$\beta(s) \Big|_{s=s_R \equiv M_R^2 - iM_R \Gamma_R} = \lambda_i, \quad i = \text{“120”}, \text{“168”}, \text{“2520”}, \text{“4752”} \quad (32)$$

with $M_R > M$ and $\Gamma_R > 0$. The loop function $G^{II}(s)$ in the fourth quadrant, neglecting the meson masses and using a common mass \hat{M} for the baryons, can be found in Eq. (14) of Ref. [55]. The equation (32) has solutions only for negative eigenvalues, $\lambda_{\text{“120”}}$, $\lambda_{\text{“168”}}$ and $\lambda_{\text{“4752”}}$. As mentioned, the “168” irreducible representation of SU(8) leads to the most attractive s -wave meson–baryon interaction, and it becomes the only non-vanishing WT contribution in the strict $N_c \rightarrow \infty$ limit.

To understand the N_c evolution, the approximated relations of Eqs. (15), (16) and (17) of Ref. [55],

$$\delta^2 \log \delta \sim \frac{24\pi^2 f_0^2}{N_c \lambda_i M_0^2}, \quad \delta \equiv \frac{M_R - \hat{M}}{\hat{M}} \quad (33)$$

$$\frac{\Gamma_R}{M} \sim -\frac{\pi \delta}{\log 2\delta} \sim -\lambda_i \frac{N_c \delta^3 M_0^2}{24\pi f_0^2}, \quad i = \text{“120”}, \text{“168”}, \text{“2520”}, \text{“4752”} \quad (34)$$

are quite useful. There exist two different situations, neglecting logarithmic corrections,

$$\lambda_i \sim \mathcal{O}(1) \Rightarrow (M_R - \hat{M}) \sim \sqrt{N_c}, \quad \Gamma_R \sim \sqrt{N_c} \quad (35)$$

$$\lambda_i \sim \mathcal{O}(N_c) \Rightarrow (M_R - \hat{M}) \sim \mathcal{O}(1), \quad \Gamma_R \sim \mathcal{O}(1) \quad (36)$$

From the results of Table III of Ref. [16], we can see that the two $\Lambda_c(2595)$ states predicted in Ref. [77] and the $J^P = 3/2^-$ $\Lambda_c(2625)$ resonance stem from the 168 representation, and thus one deduces that their widths and excitation energies behave as $\mathcal{O}(1)$ for $N_c \gg 3$, as predicted by Witten almost 30 years ago for genuine qqq states. However, the width and excitation energy of the fourth resonance in the table, located around¹³ 2800 MeV and associated to the 120 representation, grow as $\sqrt{N_c}$ in this limit. That is, this resonance would disappear, since it becomes wider and heavier as N_c increases. This behavior would be similar to what we have seen earlier in Figs. 1 and 2. Note that the large 4752 is attractive and contains many exotic states that would disappear in the large N_c limit as deduced from the above discussion.

The fact that the $\Lambda_c(2595)$ resonance survives in the large N_c limit, contradicting the findings of Figs. 1 and 2, is however quite natural. Indeed, it is natural to admit the existence of a (perhaps) sub-dominant qqq component in the resonance wave function. Indeed, this resonance has been studied with some success using a constituent quark model in Ref. [91]. Thus, one might expect the N_c behavior close to the physical value $N_c = 3$ of the resonance is non qqq due to the unitarity

¹³ This resonance, with large couplings to $\Lambda_c \eta$ and $\Xi_c K$, is also found in Ref. [12].

logs, but this sub-dominant qqq component would become dominant when the number of colors gets sufficiently large [50, 51].

It is interesting to note that recently lattice QCD simulations have started to probe the dependence on N_c of the properties of mesonic [92] and baryonic [93, 94] states. (See, Ref. [63] for a comprehensive review.) Testing the N_c dependence of the $\Lambda_c(2595)$ and other proposed molecular states can help to unravel their true nature. In this sense, the present study should serve a motivation for such studies.

V. SUMMARY

Understanding the Fock components of a hadronic state is a nontrivial task due to the non-perturbative nature of the strong interactions at the relevant scales. Recent experimental observation of the so-called XYZ and baryonic pentaquark states have challenged the conventional wisdom that baryons are composed of three quarks and mesons of a quark-antiquark pair. More surprisingly, large hadron-hadron components are predicted for certain well established hadrons, e.g., the $N(1535)$. In the present work, we have used two widely accepted approaches to qualify the $\Lambda_c(2595)$ as a dynamically generated state, namely, the compositeness condition and the large N_c evolution. Our results show that, although the relative importance of a particular coupled channel cannot be determined in a model independent manner, the basic picture that the $\Lambda_c(2595)$ has relevant meson-baryon components emerges as a robust conclusion. We have also shown that the commonly defined compositeness of the state depends on the included coupled channels, and also on the scheme adopted to renormalize the ultraviolet divergent meson-baryon loop function, which appears in the unitarized approaches. The importance of the molecular picture is also corroborated by our study of the dependence on the number of colors of the mass and width of the $\Lambda_c(2595)$. It is shown that for moderate $N_c > 3$ values, they differ largely from those expected for a genuine qqq state. We can not however discard the existence of a (perhaps) sub-dominant qqq component in the resonance wave function, which would become dominant when the number of colors gets sufficiently large.

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